JÁNOS BOLYAI'S (1802-1860)

NON-EUCLIDEAN GEOMETRY AND HIS SEARCH

FOR ULTIMATE REALITY AND MEANING

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Revised version

OUTLINE

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1. INTRODUCTION

For about two thousand years, the Euclidian system was seen as the model of admirable certainty and consistency. Descartes, for instance, in his 'Meditationes… V.' (1641) included the stable 180° sum of the angles of the triangle as one of the proofs of God's existence. However, already from the time of the Greeks inconsistencies were discovered and in the early nineteenth century it became clear that the system cannot be proven. The passionate involvement with these problematics may suggest an absorbing, numinous and even Ultimate Reality related attraction. Patterns of personal experiences, abstract geometric formulations, and metaphysical thoughts overlap and tend to stick together. The 'qualitative' and patterned structural nature of geometry may offer analogies to real life situations which can be expressed through geometrical analogies as, for instance, 'going in circles', 'squarehead', 'this the point', 'leave it behind', 'higher order'.

Three precursors of Einstein's relativity theory have attempted to grasp the Universe through going beyond Euclid's conceptions. They are sometimes mentioned as

providing the Copernican revolution for geometry. János (Johannes) Bolyai (1802-1860) in Transylvania was the founder of non-Euclidean 'absolute geometry' (Scientia spatii …', 1823; published as Appendix', 1831; Responsio, 1837). Subsequently, Nicolai I. Lobachevski (1793-1856) in Russia (1826), Carl Friedrich Gauss (1777-1867) in Germany (1855), and others published their own non-Euclidean geometrical systems. (Gauss published his work after reading Bolyai, Lobachevski and the others.)

Bolyai's life and thoughts were explored by several scientists and dramatists. My main sources are József Suták (Su.), the Hungarian translator and editor of the 'Appendix' (1897) ; Ferenc Schmidt (Schm.) biographer, for the introductory part of the 'Appendix'; psychoanalyst Imre Hermann (He. 1945, 1855, 1980, 1981) in Budapest; Tibor Weszely (W. 1974, 1981, 1988), mathematics professor in Bucharest and in Turgu Mures (Marosvásárhely), Bolyai's home town; András Prékopa (2002) at the Hungarian Academy of Sciences, and Elemér Kiss in Turgu Mures (1994, 1999, 2000).

Here I attempt to present the life and the thoughts of János Bolyai for whom, as for many other geometricians, geometry was a central life experience and the way of authentic self expression.

2, LIFE COUR99SE

2.1. Education.

János Bolyai was born on December 12, 1802, in Kolozsvár (Cluj) in Transylvania, the eastern part of Hungary, and was the very carefully educated, at that time only

son of polymath mathematics and physics teacher, and by today still noted philosopher, Wolfgang Farkas Bolyai. Until age nine, János was instructed at home through playing and conversation. His father was his main teacher.

He was a prodigy. At the age of four (Farkas' letter to Gauss in 1807; Schm. XXI.) he wanted to know whether Jupiter is not very far, since it can be seen from the city as well as from the country. At five, he cut out the 'sinus' from an arc of potato peels. And up to his adulthood János was fascinated by the 'point', a 'firstborn' child, as himself. The point is in the 'same person' both a child and a parent as the 'mother' of the 'sphere'. At 13, János substituted his father in teaching higher mathematics and geometry; and the students were more satisfied with him than with his father.

János liked to play with the other children. He had short-lasting tempers, was irritable and restless in his adolescence, but he was very good-hearted and noble-minded. He was simultaneously introduced to the reality of fragmentarity in life through his 'hysterically nervous' (later psychotic) mother, and to the contrasting amazing order of the universe taught him by his father. Farkas regularly reported about his son János' development to his friend Gauss during their correspondence period, and he was asking Gauss for advice.

At the age of 16, in 1618, János entered the military Imperial and Royal Engineering College in Vienna. He was a student they were proud of: he was the very best in mathematics, in playing the violin and in fencing. He played the violin in the most elite quartet in Royal Vienna. Once he substituted for a concert violinist, and with success. He also excelled in chess, and in languages.

2.2. Adult life.

From 1820 on, in the years when his mother was very sick and she subsequently died (1821), János started to deal with the parallels, although his concerned father vehemently tried to dissuade him from dealing with this 'bottomless darkness' which 'can swallow even a tn mmnhousand of Newtonian giant towers' (Prékopa, I. p.17.). In 1822, at twenty, in Temesvár, his first post as of a military officer, János invented non-Euclidean geometry. In 1823 in a letter to his father, he announced with joy that 'I have built a new, other world from nothing'. Then he wrote a paper in German and sent it in 1626 to his former mathematics professor, later general, Johann Walter von Eckwehr. Subsequently, he formulated the full text (147 pages in the Hungarian translation) and a brief version in Latin in 1831, the so called *'Appendix'* which was responded to with a psychologically rebuking letter by Gauss.

In the military, by 1832, Bolyai was a captain. This was the year when, on April 1, Gauss' letter claiming priority and praising himself arrived. After this event Bolyai was not able to regularly work any more. He already had malaria in 1826; and he just barely recovered from the Europe wide epidemic cholera in 1831. He lost his usual stamina to resist stress. He lost interest in his assignments and was asking for early retirement. The very popular and sociable person who was qualified by his superiors in 1931 as 'taciturn, communicative, friendly, and respectful toward either superiors, peers or subordinates, or in the community', was described in 1832 as 'irritable', 'sensitive', and 'withdrawn' (W. 1981, p. 26.). It can be noted that the suicide of 27 years old Niels Abel was attributed by the Norwegian scientists to Gauss' claiming priority for his not published knowledge, and that some former friends of him, like Jacobi, and Legendre too, claimed that they were treated in similar way and were in uproar (W. 1981, 29-30.).

Returning home, Bolyai mostly lived in misery on a limited pension. He lacked recognition, and he had no access to new books and periodicals. In order to publish

in the West he would have needed the support of a respected mathematician like Gauss, but the Bolyais did not know any other great mathematician to dare to contact (W. 1981, 26.),

He started a family and to provide for them, he managed a small property in the country. However, for long years he had to live common law with his wife in the legal relationship of employer and housekeeper, because they could not afford to put down the caution-money required for a military officer to marry. János later left the property to his wife and the two children, and they departed.

It is sad that, through the years, he got into mutual criticism and rivalry with his father. Still, János always spoke about his father with highest respect, and he cared about him. When Farkas became very sick, János informed the family about his condition. It would appear that Farkas may have resented that his son surpassed him in the two most important fields of his life, mathematics and marriage. Namely, widowed Farkas was courting János' later wife, but the young woman became acquainted with János and she fell in love with him. (Farkas later married an other woman from whom János' half-brother, Gergely, was born.)

Bolyai died in 1860, at the age of 58, of 'pneumonia' and 'meningitis'. The country people spoke about him as a silent, sickly and kind-hearted man, but the prejudiced and conventional city people created humiliating rumours about his strangeness and cynicism. Apparently, he deliberately withheld the expression of his intimate affects before public eyes; but he remained nice to his children.

2.3. János' father, Wolfgang Farkas Bolyai.

János' father, Farkas (1775-1856) was a mathematics and physics teacher in the College of Vásárhely (Marosvásárhely; Turgu Mures). He may have been even more a prodigy than János, and he was a real polymath. In the Encyclopaedia Britannica and in philosophical dictionaries (i.e. The Cambridge Dictionary of Philosophy, 1995; from now on Cam.) he is mentioned as mathematician, poet and dramatist. He made agricultural, pharmacological and heating industry innovations. He was a popular fireplace builder. He wrote five dramas, published in forestry, and made an altar painting. In education he followed Rousseau and consequently until the age of nine János did not receive formal education. Farkas also was a great patriot, and he was extremely democratic. He made his own bed and did not accept personal services from his valet. In 1815 Farkas persuaded the aristocracy to make a fund to build a theatre in Vásárhely, for this was seen as the only possible means for adult cultural education at that time. Farkas Bolyai was a very respected person in his community.

In his two-volume 'Tentamen' (1629-1632) and his 'Arithmetica ...' (1843) Farkas Bolyai dealt with the 'foundation of mathematics'. He intended to either prove or disprove that parallels meet in the Infinite. He applied Euclid's 'L' lines in rotations, but he was not successful. However, his original ideas inspired later mathematicians, among them Riemann in 1854 and Helmholtz in 1868 (Su. XIV.).

During their student years in Göttingen Farkas became intimate friend with Carl Friedrich Gauss. Later, Gauss' picture hung on the wall of his studio. Little János was instructed that this was the picture of his father's friend who is at the same time the greatest mathematician in the world. They almost took it for granted, that János will study in Göttingen. However, from between 1808 to 1832 Gauss was not responding to Farkas' letters; and when he responded to the Appendix, he hurt them.

3. BOLYAI'S WRITINGS

3.1. The 'Appendix' or 'Scientia Spatii …

The *'Appendix'* was first published in1831 under the title *'Scientia Spatii Absolute Veram Exhibens: A veritate et falsitate axiomatics XI Euclidei (a' priori haud unquam decidenta) independentem; Adjecta as casum falsitatis quadratura circuli geometrica.* It was a 27 pages long shortened Latin version of the original German manuscript (147 pages long in the later Hungarian print). It was attached as the 'Appendix' to Farkas' 'Tentamen...' in 1832. Gauss only responded to the second sending in 1832. He stated that he could not praise János' work, as he would praise himself (Urquhart, 1999; Prékopa, 2002, I, p.8.; all biographers), because the way and the conclusions correspond almost completely to his own reflections. He worked on them for 35 years, only he had not published them. He is 'glad' that the son of his old friend has preceded him in publication.

Gauss hid János' work from the world, so that the Appendix was only found in 1867 in Gauss' chattel, years after his death in 1855. The importance was immediately recognized, and it was translated into French from Latin the same year and was printed in 1868 (Oláh and Zarug, 1996; Mester, 1999). It has turned out when Gauss' correspondence was published in 1859 that to a small circle of mathematicians Gauss had privately mentioned Bolyai's theory.

Bolyai discovered that Euclid's disputed Vth axiom about the parallels (which was called by some authors, like the Bolyais, the XIth), is independent from Euclid's other postulates, and that therefore it can not be proven through Euclidean geometry. A more universal, 'absolute' form of space theory is needed. Bolyai demonstrated

that *in a hyperbolic space an infinite number of non-intersecting lines* can 'burst out' from one single point, and *infinite numbers of spaces are possible*; the traditional three-dimensional Euclidean one being only one of them. The trigonometry of the sphere, and other theses are independent from the hypotheses about the parallels; these are the constituents of absolute geometry (Appendix, 32.). *New spaces emerge* when these theses are modified by an uncertain and freely chosen constant "i" or by some other 'signs'. However, *the system itself does not change*. (Lobachevski in his similar conception applied a constant "e" sign. Both "i" and "e" are signs is Euler's equations. Leonhard Euler (1707-1783) has called the square root of '-1' "i", as a shortening for 'imaginary'. Later, Rieman wrote that the new condition arises through a "contract".

For his calculations Bolyai also applied, among others, Descartes' analytic geometry, and Newton's and Leibniz' differential and integral equations (Prékora, II, p.6.).

Gauss in his later comments on Lobachevski between 1840 and 1848 relied in part on Bolyai's Appendix (Prékopa, II, p. 5.).

3.2. The Responsio. Treatise on the Imaginary Quantities.

In the `Responsio' (1837), a paper which was not recognized at a competition in Leipzig, Bolyai explains the implications of absolute geometry through the *theory of complex numbers. These 'signs' can change the whole space-world and can create either Euclidean or non-Euclidean hyperbolical spaces.* They are valid to elicit changes in the real world too.

The Motto of the Responsio is: „Fructus nonnisi maturi decerpendi". 'Only the ripe fruit is allowed to be picked.' (He. 29). This may appear as bearing the symbolic

meaning of completed achievement and the resulting independence from one's parents, like his possessive mother, the instructing father, and Euclid as well. It may have meant doing something adult-like, important and responsible, such as bringing forth a perfect system, a ripe fruit. (The symbol of the point as both child and parent is revitalized. The 'child' 'fruit' has matured and it has become 'parent' to a new child, a new geometric system.)

As for many other geometers, for János Bolyai too the abstract scientific procedures have symbolized emotionally important experiences. We can recognize in the Motto a conflict with János' overwhelming conscience: he has to justify himself as a grown up deserving permission to guiltless independence and autonomy, deserving forgiveness for leaving his sick mother (becoming a parallel), and for surpassing others, particularly his father, through daring to make an extraordinary achievement.

Regarding the 'family' symbol in the biological-geometrical-logical-social-spiritual implications (the last one being conscience related) of the Motto of Bolyai, it has to be remarked that analogies to the genealogical tree were commonly used in the everyday language of the Bolyai household. (For instance, János referred to himself as to a 'first born' point.) And in the non-Euclidean system, the genealogical structural model can be extended to the whole Universe (to the infinite numbers of deriving spaces and worlds), as this was indeed implied in both Bolyai's and Lobachevski's metaphysical epigenetic philosophy and influenced their views on ultimate reality. The logical seed is the characteristic *'pluri-univocal'* relationship of the genealogical tree (described in four equations by Serrus in 1945) where the same person can be mathematically identified as being simultaneously parent, child and sibling, and where there is also an 'one parent to many children" and vice-versa relationship.

3.3. Manuscripts:

There are 14.000 pages largely unpublished manuscripts and correspondence by Bolyai which in his later life he ordered and numbered. Through several decades, leading Italian and French scientific institutions made efforts and offered to translate Bolyai's manuscripts assuming that they must contain some further important insights and discoveries. However, the local authorities resisted handing them over. This was, probably, because of some difficult to understand, and perhaps even psychotic style passages (Vekerdi, 1981). At the present, the editing of the manuscripts is in advanced processing since a start by the late Samu Benko, and is continued by Elemér Kiss, Tibor Weszely, and other researchers. A full presentation of this progress is beyond the scope of the present study.

3.3.1. Mathematical topics

There are further reflections on mathematics in Bolyai's writings, and he also prepared a 'Completion' to the Appendix. Other topics were – among others – cubing the tetrahedrons; freedom of contradictions; observations on Lobachevski (Raumlehre); and number theory which latter has become a textbook material worked out by Jeans, 38 years after Bolyai's work (Prékopa, III. p.4.).

3.3.2. 'Üdvtan'

About 2000 pages of Bolyai's manuscripts constitute the *'Üdvtan'* (Science of Social Welfare, or Science of Human Happiness, 'Heillehre' in German) which Bolyai started to write in 1819, at the age of 17. It is a 2000 pages long manuscript.. He

worked on it simultaneously while dealing with the problematic of the parallels. From the simultaneity and from Bolyai's different comments, such as his announcing to his father that his geometry is a contribution 'to the welfare of Mankind', we may conclude that the Appendix and the Üdvtan are two related parts of the same encyclopaedic opus to promote general social welfare.

Bolyai's preoccupation with 'justice', moral truth, in addition to searching for the abstract objective 'truth', was extremely actual. In 1816-17, just before he left for Vienna, there was a famine in Transylvania. In Marosvásárhely alone, several persons died. Bolyai wanted to build a 'new' and better world to replace the 'lost paradise'. His focus was particularly on education and economy. (He was somewhat influenced by Saint Simon, Fourier, and Owen.) His idealistic intentions may have been fuelled through the comparison of the famine and the intellectual misery of his local city with the far better life in the rich and culturally advanced West, and through his own experience that he was rescued by his father from the chaotic life at home with his mother.

He thought that *only a scientific foundation can bring order into life.* This is, because the world is an 'organized whole' (He. 61.) where everything is rigorously interconnected and is of equal rank, so that an overall encompassing rule has to be observed to serve the building of a better society.

His views about 'nature' corresponded to those of his time: post-Enlightment, post-French Revolution, pre-Romanticism, post–baroque, and the *epigenetic* Rococo views which latter flourished in Lobachevski's Russia as well. The epigenetic theory was originated by William Harvey (1578-1657) who also discovered the blood circulatory system, and it was followed by some greatest embryologists, like Kaspar Friedrich Christian Wolff (i.e. in 1768), von Baer, Johann Wolfgang Goethe and

others (Funk Encyclopaedia, 1979; Vill and Ariel Durant, 1965; Imre Hermann, 1945). Harvey distinguished two processes of embryo formation: „metamorphosis", the well known continuous process, like plants from the seed, or chicken from the egg, and *„epigenesis"* where *development occurs only through the addition of new parts.* Through particular additions *heterogeneous units of equal rank* can develop from a homogeneously amorphous common arch-formula or from a single cell, like all the different organs of a plant may develop from an original leaf-like formation. This is possible because each germ contains within itself the germs for many future descendents.

The epigenetists denied deterministic evolutionary preformation. Bolyai's application of the modifying creative „i", and the assumption of an unspecialized amorphous arch-substance or arch-mother, corresponded to the structure of these discoveries. The idea was vaguely present in Aristotle's worldview as well. Bolyai referred in the 'Üdvtan' to *man's 'original first nature'* which *can be transformed*, and may „instigate" us to bring forth changes to pursue the happiness for both the individual and the community. According to Bolyai, *man can do everything, the only limits are the laws of nature.* He considered that non-Eucledian geometry reflects the general natural law which has to be observed, and that, therefore, geometry is a contribution to promote the welfare of mankind.

Bolyai was clear that the education of mankind can be realized only gradually, across several generations; i.e. 12 teachers could educate 144 good teachers, so that the number of educated persons in the community would increase logarithmically. His general idea was to make a *class-free society* realizable. He can be seen as representing a pre-Marxist view.

Bolyai tried to reform language as well. He took part in the national movement to reform his Hungarian mother tongue (Molnár E., 1975, Sarlóska, E., 1973). For instance, he translated Latinized words into Hungarian, i.e. aesthetics into ’széptan’ (study of beauty); but he also experimented with constructing a ‘perfect’ ‘universal’ world language for which, in a pioneering way, rules from music theory were applied. He wrote a chapter on music theory.

3.3.3. Autobiographical notes

In his unfinished Autobiography Bolyai writes with an amazing sincerity about his childhood, his loves, his interests, the values he pursued, his political stance, his relationships, and his relationships and opinions with regard to his father, his late born younger brother, Lobachevski, and Gauss.

A few examples:

From his childhood experiences he recalls that already at the age of 3 or 4 he was inclined to “independent thinking”. He wondered about the essence (“lényeg”) of God, and had ideas about the “infinity” of space. He liked to draw mathematical forms on the floor with a chalk, and he was eager to study the sky. Further up to his adulthood he was fascinates by the ’point’. The point, a “first born” child, like himself, is “in the same person“ both a child and a parent. The same “child” point is the “mother” of the sphere and it symbolizes the arch-mother of the universe.

From his young adulthood, Bolyai narrated about fencing competitions, and about having a few duels. He himself was the provoker only once, and this was when he was a student. Before it would have become serious, the watchers separated them

and let them make peace. He was and remained in friendly relationship with all his companions both before and after these acts. (W. 1981, 39.).

About his character generally, at age 43 (1845) Bolyai reported that *„there were two main dominant, basic trends in my whole life: unlimited love for the truth (in doctrine and in morals) and unlimited respect for the woman gender". "The first is pure virtue, the second is in part merely nature; but I was often involved in tender endearment"* (W. 1981, 39.).

Bolyai was enthusiastic about virtue and truth ("igazság"). His well-achieved goals were faultless thinking and objectivity (fairness) in either scientific or social behavioural matters.

His personal involvements, like the particular respect for the woman gender ("a némberek") were quite complex:

1./ It was indeed not merely sensuality. János was in the military, and he was still a virgin (father's letter to János). He had one great love when he was already an officer. He was not noted as adulterous while he was married and the psychoanalyst biographer is not suspicious of sexual exaggeration either. Only in his later years, when he lived separated and alone, had he contacts with different women. And he writes about the unlimited joy from mathematics which is 'purified from sensation'. Therefore, as he describes it, his main wish would have been for a peace loving, kind and committed life companion who would accept her partner's (difficult, impulsive) personality, and who would expect the same understanding attitudes from the man partner. Basically, János was longing for a motherly wife.

2./ A component of János' romantic attractions came from his insecure attachment and the wish for compensation of the deprivations and conflicts caused

by his mother who was talented and loving, but also possessive and depriving, not fully normal. In Hermann's consideration a 'negative' Oedipus complex (wish to leave mother) and an intensive *'separation conflict'* which started in János' childhood has become a permanent preoccupation in his adult life, and this has influenced the style of his geometrical thinking. The particular separation experience when leaving for studying in Vienna was symbolically alluded to by János in the first sketch of the 'Scientia spatii...' from 1820 (He.1945, 35.). He named the first parallel line (as his system allowed an infinite number of them) differentiating from the other lines drawn from a chosen point, as a *'bursting away'* ("elpattanó") line as if it had symbolized his 'bursting away' from the overly possessive mother. (Half-orphan Lobachevski, on the other hand, fantasized about this line as a compensating 'touching' line.) Hermann connects this 'bursting away' to the personal experience of the acute, but subsequently eternal separation from his sick and soon dying mother. Another symbol of the bursting away tendency suggesting guilt and the hope of forgiveness, the idea of the 'ripe fruit' to be picked off only with sufficient justification, had such an importance that it became the Motto of the 'Responsio'.

3./ 'Love' and 'enthusiasm' appear as central in Bolyai's spiritual experience, Since the whole universe has an analogy to the mother-child relationship, romantic feelings through a 'warm-hearted' woman may have represented a soothing and *revitalizing contact with a cosmic arch-mother* as well. We can sort of witness an intricate motivational fantasy world where there was longing for János' real mother, for a warm-hearted normal woman partner with sexuality and affect, and for the harmonizing and spiritually satisfying cosmic parent which János was able to contact through his mathematical work.

Further in the Autobiography, Bolyai recounted his *'political activities'*. For example, he narrated an event after the oppression of the national freedom fighting in Hungary

in 1848-49. A high standing general treated him 'very nicely', and offered him an opportunity to become a political informant. Bolyai did not take that job to improve his life condition (W. 1981, 50-51.).

Relating to *Gauss* (W. 1981, 14.): Notwithstanding the disappointment and harm Gauss had caused him, Bolyai thought about him with forgiveness and understanding. After the death of Farkas, he reflected about how things had happened as they happened. It is likely that for over twenty years Gauss did not answer his father's letters in which, finally, Gauss was asked to take the son to himself as his student for three years, because he did not want to either accept or refuse Farkas' offer. It is no wonder that Gauss was „reluctant to teach"; but telling this with a straight refusal would have been oppressive ("nyomasztó") or, at least, very unpleasant ("kellemetlen") for their relationship. They were old friends, the two "Colossi". As it turned out, it was better for him that he was guided by his father. Gauss probably never would have inspired him to such enthusiasm and, particularly, to the intensive love of mathematics and of philosophy. Nor would have Gauss assisted him with the „self-education of the very best parts" of his personality.

Bolyai remembered that, when he bought Lobachevski's very similarly conceived book (that from 1640), his first impression was that underhanded Gauss gave over the Appendix to Lobachevski. But as soon as he read it further, he realized that this did not happen; and Bolyai recognized with great respect that Lobachevski had made an original contribution which was similar to his.

Bolyai excused his father, too, who unjustly favoured Gergely, the son from his second marriage, giving him János' maternal inheritance from his first marriage, and this he did in a time when János and his family were in need. János later resigned legally from this property; and he remained quite supportive and loving towards Gergely.

Based on his autobiographical reflections, Bolyai appears as having been a great man, not only a great geometrician. Notwithstanding his inclination to short-lasting rages and to exaltation and strangeness, when it came to decisions in important matters, he was realistic and composed. He had full insight into his own mistakes and was ready to correct them. Mature man Bolyai's main attitude was wisdom, responsibility and generosity, even though in his outer behaviour he sometimes showed cynicism. A touch of this latter was observed by a visiting mathematician in his aging father, too.

It would seem that suffering and frustrations have hardened Bolyai's formerly flexible personality. Still he has maintained his final optimistic hopes. The Autobiography reflects the *experience of harmony* and the belief in man's ability to be transformed and to build a 'new world' which will 'burst' into an autonomous life; and it also shows trust in the devotion of the creative individual and, in Bolyai's term, in "Providence" (the underlying arch-law for the 'bounded freedom' (He. 1945, 57.) as well. Bolyai's abstract philosophical system corresponds to his experiential personal life and to the *tacit faith in an 'absolute' Parenting source who is tolerant* towards all his potentially infinite numbers of children, and who generously shares his power.

4. THE RELEVANT HISTORY OF GEOMETRY

4.1. Euclidean geometry

Euclid was a disciple in Platon's Academy. His *'Elements'* from around 300 B.C. was known in the West mainly through an Arabic transcription printed in Rome in

1574. Euclid worked out the *'foundation of geometry* which is seen as a branch of mathematics together with number theory, calculus and mechanics. Euclid himself differentiated geometry, the study of continua from mathematics, the study of discrete entities (Cam. p. 594.). The Bolyais considered geometry as a part of mathematics.

Euclid worked out five 'self-evident' starting-point theses. These he called *postulates* (needed assumptions) and axioms (statements) which terms are used today in rotation. Some of these postulates deal with the issue of the parallels. According to the 23rd definition in the 'Elements', *parallels* are those lines in a plane which never meet, not even when they are infinitely prolonged. In the Euclidean system from any point in a plane only one parallel line can be drawn to a given line.

4.2. From Euclid to non-Euclidean geometry

4.2.1. Critical Studies.

Already Ptolomaeus in the second century and later Geminus, and Proclus (A.D. 410-485) discovered that straightforward clear proof for the Euclidean parallels theory was missing. The knot of the problem was the puzzling axiom about the parallels in the Eleventh or *Fifth Postulate* which stated that if two parallel lines are crossed by a third one, the two lines will meet on that side of the third line where the sum of the inner angles is less than 2R (180°). It would follow from this that the sum of the angles of the triangle in a plane is 180°. Since we cannot generate empirical proof for this conclusion, because we cannot draw lines with infinite lengths, logical proofs would have been needed which, however, nobody was able to forward before Bolyai.

In the critically oriented Enlightment period, there was a renewed interest in the parallels. Because of the great respect for Euclid, everybody wanted to 'prove' the

validity of the fifth axiom. While beforehand altogether 55 works were written, in the second half of the 18th century 67 treatises were circulating, some of these by D'Alembert, Lagrange, Fourier, Monge, Playfair, and Laplace. Since nobody could prove the fifth axiom, gradually the insight was generalized that a different foundation should be worked out for geometry. János Bolyai was the first who did this in 1823.

4.2.2. Some of the direct precursors of Bolyai.

In 1766, *J.H. Lambert,* autodidact physicist, epigenetist, and a somewhat similarly eccentric and affectively remote, schizoid personality like Bolyai, distinguished three types of 'spheric triangles'. He also had the uncertain idea of involving complex numbers with the third type triangle.

Girolamo Saccheri (1773) and *Mary Legendre (*1794), advanced the exploration of the connections between the fifth axiom and the sum of the angles in the triangle.

In 1804, *Farkas Bolyai* worked on a *'Theoria parallelarum'* (Su. IX.) which he sent to Gauss. Here and in his other works he may have given some useful hints which may have been picked up by János.

In 1819, based on his correspondence with Schweikart, Chr. Gerling wrote to Gauss that a new foundation is needed for geometry (Su. IX.). Gauss agreed as this was his idea too. (Gauss tended to agree with his correspondents stating that he himself had come to the same idea.) In 1824 Gauss wrote that a 'non-Eucledian' geometry should be worked out. He did not know that one year beforehand Bolyai had already worked it out. According to Bolyai in 1832, *„Gauss is the only one who made a few steps forward to the goal, but he was still far from seeing all these in a whole"* (Draft of a

letter written in Lemberg to Archduke John, chief of the military, May 18, 1932; W. 34.).

4.3. János Bolyai's contribution.

According to *A. Urquhart,* 1999, Bolyai, and soon after him Lobachevski, had the 'courage' to publicly state and demonstrate their convictions. Bolyai was the first in working out a *non-Euclidean geometry system*, and he established the principles of *absolute geometry which encompasses any geometries.* Further, he decided that since the implications of the *fifth Euclidean axiom cannot be proved, there is no necessity to rely on it.*

Non-Euclidean geometry was the first contradiction-free closed mathematical system which had no direct connection to reality. Further, Bolyai revolutionized scientific problem solving technicality through introducing interconnected, general axiomatic thinking (Prékopa, III, p.7.). Later, David Hilbert (1862-1943), receiver in 1910 of the second Bolyai prize, conceptualized a fully abstract exact axiomatic system of space with an infinite number of dimensions.

In its short term historical consequences, the non-Euclidean system appeared at first as conflicting with *three-dimensional physics* and as lacking the possibility of practical application since it allowed an infinite number of spaces. It was Einstein who established that nature can be understood through non-Euclidean geometry only. As reported above, Bolyai did not see any dichotomy between the Euclidean and the non-Euclidean systems, and he was able to encompass Euclidean geometry as having a place in absolute geometry. The by some so assumed incompatibility may remind us of the heated, but for today resolved debate between the representatives of

traditional science and physics and the emerging post-Heisenbergian and postmodernist views (i.e. Mester, 2004).

Further also, while *absolute geometry has endorsed both incompatible views, the Euclidean and the non-Euclidean,* it contrasted with the *Kantian* picture of a single a priori science of space and of closed order. The Kantian view was confronted by Helmholtz and others.

According to Imre Hermann (1945, 79.), the Appendix was indeed a kind of revolutionary step, and not only against Euclidean geometry, but 'unintentionally' though, somewhat also against religion, because Bolyai has weakened the Euclid based Cartesian proof of God.

Euclidean geometry has become associated with linear, and non-Euclidean geometry with non-linear thinking (i.e. D. Schuldberg, 2000, Art. 41.; Cam. p. 537; Hermann). In Hermann's view, open ended and dynamic, non-linear 'absolute geometry' is congruent with a natural 'anarchistic' state. In sum, a first wind of deconstruction and pluralistic post-modernism started to blow with the debates on non-Euclidean geometry, albeit Bolyai himself saw his work as *completing and integrating*, and not as denying existing scientific tenets.

Bolyai also expressed insight into the *principle of relativity theory* 100 years before Einstein's mathematical formulation: "The law of gravitation is strictly connected and it appears in continuation with the stature (termet), the constitution (alkat), and the structure and nature of the space, and with the flow of the world ('a világ folyásával'; W. 1993).

4.4. Post-Bolyaian development.

4.4.1. Lobachevski (1793-1856)

While Bolyai was strictly theoretical, subsequent authors were more inductive and empirical.

In 1826 Nikolai Lobachevski, long time rector of the University of Kazan, then suddenly discharged for political reasons, presented his work first in French. In his 'imaginary geometry' the value of the imaginary 'e' has to be established through observation. His system has 'limits', the straight line is substituted by a special 'bordering circle'. The laws of physics inhibit the development of a free geometry except for when mechanics itself changes as in Laplace's demonstration. Lobachevski concentrated more on trigonometric geometry than on absolute geometry. He did not consider the role of gravitation.

Lobachevski lost his father at four. Growing up, he was at first revolutionary, then restrictive and adaptive. He too was interested in epigenesis and he too was enthusiastic about the development of mankind with education through the series of generations from the primitive ferocious manifestations to the enlightened state.

4.4.2. Models by Schweikart, Beltrami, and others.

A. F. Schweikart who dealt with a 'new' 'astral' (hyperbolic, planar, celestial) geometry in 1819, has taught a similar geometry to Bolyai's in 1826.

Later in 1868 *E. Beltrami* produced a model of a Bolyai-type two-dimensional space inside the planar circle to prove the system's freedom of contradiction; and he pointed out the logical dependency of the non-Euclidean geometry on the original Euclidean system.

Jules H. Poincaré (who received the first Bolyai prize in 1905), *Felix Klein* in 1872, further, *Fuchs* and others, developed fermenting models which contributed to the emergence of new branches in mathematics (Prékopa, III, 2., 5.).

4.4.3. Gauss.

Already between 1808 and 1816, or perhaps even before, *Gauss* came to his break with Euclidean geometry. He founded the 'new school of mathematics' with Jacobi and Dietrich (He. 1945, 64.) which aimed to find a 'main idea', or common law, to encompass several other ideas, as well as individuals and communities of equal right. Gauss himself was working on a non-Euclidean geometry when he received János' book. Further up we have quoted Bolyai's opinion about Gauss' contribution.

Gauss prevented Bolyai's recognition through hiding his book from the eyes of the public. However, through his rebuking self-praise he indirectly complimented him, and in some of his private letters, like that to Gerling, Gauss mentioned that Bolyai's work is a 'great asset' and that *Bolyai is a 'first rank genius'* (Su. XIV.; Schm..*)*. Still, even in 1846, when he publicly recognized the famous councillor of state from Kazan Unversity, and also Schweikart, he did not mention Bolyai (W. 1981, 29). - Gauss' own non-Eucledian geometry which he published in 1855 was not found to be faultless.

4.4.4. Riemann, Helmholtz, and Einstein

The thinking about curved space has led to the emergence and the articulation of *Einstein's relativity theory* (Cam. 537.).

Thirty years after Bolyai, former student of Gauss, *Bernhard Riemann* in Göttingen (1854) established his own system. He explained that if the notion of parallels is excluded, the lines will always meet, and the sum of the angels in a triangle exceeds 180°. Through this step he completed the principle possibilities of absolute geometry: Euclidean geometry (180°, one parallel); Bolyai's geometry (< 180°, any number of more parallels); Riemann's geometry (>180°, no parallels). Also, in a talk, Riemann suggested that a four-dimensional geometry is possible, and that it would make much more sense than the three-dimensional Euclidean one. He also recognized that non-Euclidean geometry could be applied to study the cosmos and physics (Ferris, 1997). Riemann did not conclude to gravity either.

In his view of the world, similarly to Bolyai, Riemann himself postulated an originally unrestricted, unbound state.

Riemann was a frail and painfully shy man who lived in poverty and died of tuberculosis at the age of thirty one. From this point of view, his fate was somewhat similar to Bolyai's. (It may look as if 'Ultimate Reality' had an approximately 'equal rank' sad perspective for many young geometers across our history.)

Hermann von Helmholtz (1821-1892) was a physiologist, physicist and trained physician, who discovered the conservation of energy. He enfolded non-Euclidean geometry to its 'peak'. He also made great contributions to the epigenetic approach. In 1870 he attacked the Kantian view of geometry. Helmholtz inspired *Poincaré* and the logical empiricism movement, and he made Einstein's use of non-Euclidean geometry possible (Grattan and Guinness, 1995; Cam. 251.).

Bolyai's geometry was known to *Albert Einstein* (1879-1955) through his Hungarian fellow student, *Marcel Grossman,* at the Zurich Academy of Politechnik. Einstein made the so called *Riemann-Bolyai theory* the basis for his reflections.

4.4.5. Penetrating general culture

András Prékopa (III, p.6.) lists the applications of non-Euclidean geometry beyond its own field. Non-Euclidean geometry was applied in relativity theory and static physics, in communication, like for the graphics of three-dimensional manifolds (i.e. for the Internet), and in the arts. From the advancements here is only mentioned that the great Dutsch graphic, *Maurits C. Escher* (1898-1972), applied Poincaré's hyperbolic constructions for his 'Circle Limit' engravings, and *Conrad Pelthier (*1996) in Berlin applied the 'hiperboloidon' described by W.J. Reinolds. In Berkley, California, the MSRI let construct a *mathematical sculpture* based on Felix Klein's quartic curves: 'The eightfold way'. The inscription refers to its Hungarian inspiration.

5. STUDIES ON EMINENT PERSONALITIES

5.1. General findings.

Early research on creative personalities tended to differentiate two main groups of creative personalities (E. Winner, 2001). There are the 'experts' according to Kuhn's terminology innovators, traditional artists and 'paradigm preservers', and there are 'geniuses' who *'disrupt and ultimately remake a domain'*. Today most researchers drop the distinction and the term 'genius', and simply concentrate on the study of 'eminent personalities'.

Characteristics for these eminent personalities are a complex pattern of deep ’commitment’ and industriousness in addition to specific and general ’talents’. (This was already János Bolyai’s opinion.) There are, further, some apparently non-related particularities which may influence the accomplishments. For example, more than average occurrence of deprivation, often orphanhood, was found in the eminent persons’ childhood together, however, with simultaneously received compensating protection and inspiration by a devoted other adult (Goertzel, 1987). János Bolyai shared the life situation of this group in so far as he had a hard to tolerate ’hysterical’ mother, and an extraordinarily caring and instructive compensating father.

5.2. Crossing disciplinary boundaries.

5.2.1. Multitalented personalities.

Many great thinkers and artists acquire information and practices from different modalities so that on all these different fields they may excel. They also can cross-fertilize their skills i.e. through *discovering isomorph patterns* and ’analoging’, ’modeling’, or ’synthetizing’ (Root-Bernstein, 1999). As already mentioned, János’ father, Farkas, was a polimath. János, too, was not only a mathematician, but had other talents, like in languages, or music and fencing, as well. And he also perceived the analogies between family structure, geometric and metaphysical relations, and everyday manifestations.

5.2.2. Scientists and music

Great scientists, encyclopaedist Denis Diderot, Niels Bohr, Max Planck, Louis de Broglie, the American G. Davisson and L H. Germer and others applied musical analogies like the mathematically described rules of vibrating strings for mathematical explanations in biology or atomic physics. Music data were introduced to the understanding of 'musical DNAs' (Root-Bernstein, 1999, 341.). Some of these scientists were performing musicians as well, as János Bolyai was. Max Planck had seriously considered the career of a pianist, de Broglie was an eminent amateur violinist, and Einstein was 'better than an amateur' violinist. Bolyai worked on music theory as well and his view of creating new worlds or systems in geometry changing the key 'i' may correspond to both epigenetic analogy and to variations on a theme in music.

5.2.3. Biology and geometry.

Already Descartes was connecting biology and geometry. We have seen that non-linear epigenetic theory was perceived as isomorphic wirh non-Euclidean geometry.

5.3. Symbolic use of geometry.

Imre Hermann pointed out that geometrical forms and relations are suitable to symbolically express psychological meanings and may elicit either ***healing or depressing*** effects. He refers to reports by different geometers, and he quotes passages from Farkas Bolyai's use of geometrical analogies to express intensive feelings and thoughts about parents, children, birth, giving birth, feeding. In the 'Arithmetica', Farkas establishes that "in geometry we progress through continuous births" and in one of his letters he called 'mathesis' his 'wife' (He. 1845, 22., 31.).

Geometry may offer frustration, as well as a delightful order when eliciting the experience of conflict resolving harmony. Bolzano (He.1845, 14.) reported that his depression was cured through reading Euclid. Geometrical relations, like the meetings and crossings of lines may equally express ‘touching’ (Lobachevski), ‘bursting’ (Bolyai), frustration or fight (Coleridge). Coleridge saw ‘swords pointed against each other’ in the crossing lines. Farkas Bolyai and Gauss agreed in their correspondence from 1804 that the study of the parallels is like a ship shattering stone in a stormy sea (i. e. Gauss: ”... der Stein des Anstosses .... Gruppe von Klippen, woran meine Versuche bisher scheiterten”; He. 1845, 30.). Farkas repeatedly mentions that the fascination with the parallels may lead to ‘madness’ (1819; He. 1845, 30.).

It would seem therefore, that the ‘drama’ of the parallels may have represented hope for integrity and a ”better world” for some, a seducing ultimate reality for disappointed others, and a unique joyful fulfillment experience of the ability to create another “better world” for the successful ones, like János Bolyai.

We may reflect about that through the emphasized inclusion of Euclid’s geometry into his new system, Bolyai may have expressed his gratitude towards his ‘ancestor’ father, and towards Euclid and the other mathematicians. Successful János may seem to have found more harmonious enjoyment and consolation through geometry than his sometimes frustrated father.

5.4. Afflicted families.

5.4.1. Adaptive pathological trends

Although impulsivity, sensitivity, alienation and unconventional thinking to a degree of severe illness would preclude creative activity, it has been suggested since Plato's and Aristoteles' time that psychopathology and greatness are associated. Both Gauss and Riemann had 'hypochondriac', depressive periods when they were unable to work, and manifestation of a *'schizophrenic spectrum'* were found in Nietsche, Wittgenstein, Newton and János Bolyai. Nobel prize winner game theorist, John Nash (1994), was a hallucinating chronic schizophrenic (Nasar, 1998). It is suggested that a 'touch' of 'mental illness or an alternate condition' belongs to the genetic make up of some highest achievers. (Special issue of the *Bulletin of Psychology and the Arts,* 1998; ref.: 'Art'.).

According to the *'Eysenck Personality Inventory* (Eysenck, 1995) a milder and still normal degree of 'psychoticism' which can be called freedom from conventional thinking is a central trait in creativity even in the cases where the person is fully normal. According to the *'Barron Ego Strength Scale'* (Barron, 1963), creative people paradoxically show both explicit healthiness, like 'ego strength', order, organization, as well as pathological 'disorder' trends. Coleridge, for instance, worked with 'controlled' 'over-inclusive' thinking.

Openness to perception to a degree of potential vulnerability together with high controlling and modifying skills and sufficient conventionality to work within a standard framework, is seen in serious achievers (L.A. Sass, 2000, Art: 36-38.). Dante with the 'plot' of the Divina Commedia demonstrated the need for psychological safety devices six centuries before psychoanalysis. As he explains to Mohammed's question in Hell, the reason for his journey to hell and heaven is to 'gain full experience'. However, because he is vulnerable, he is sheltered by his God sent protectors, Virgil and Beatrice. (For János Bolyai his father Farkas was the protective shelter, and later his grasping into logic.)

According to *Jan Ehrenwald* (1986), independently from whether there are pathological trends present or not, the crucial criterion for extraordinary achievement (as in Leonardo, Mozart, Picasso, Freud and others) is the ability to '*temporarily integrate'* right and left hemispheric brain functions and the personality resources. This means composure, mobilization of the whole personality, and the transcending of genetic and cultural programming.

Jarvik and Deckard (1977) concluded that a selective *evolutionary advantage* is manifested in the healthy relatives of schizophrenics who represent a rather high constant 1% of the general population across nations. The relatives tend to be healthy variants of the 'schizoid paranoid" or, in positive terms, 'Odyssean personality' (Dante is said to have identified with Odysseus.) These persons tend to show vocational success, certain kinds of giftedness, social eminence, kindness, loyalty and courage, and a higher rate of fertility. On the negative side, these relatives can be aloof, deceitful (which Bolyai was not), and competitive. Interestingly, the biological relatives of schizophrenics also were more often recognized in a 'Who is Who' sample for their achievements than other members of the general population. Further, a subgroup of 'healthy' indexed children of schizophrenic mothers were more creative than any other group of children (Kinney, D. K., E. Ralevski, T. Richard,. The relatives Art: 43-44.).

In case of pathology, creative 'experts' in a field were found susceptible to have affective, 'manic-depressive' type illnesses. The often bizarre 'revolutionary' persons, like the often so called 'geniuses' who make major changes in some cultural domain, seem to show more tendency to distance themselves from the 'natural evidence' (see also St. Paul and Husserl's 'bracketing' personalities), and in this sense they are closer to the 'schizophrenic spectrum'. Because *a certain alienation is necessary* for man's greatest achievements, as in Descartes, Locke, Hobbes, Pascal,

Spinoza, Kant, Leibniz, or Einstein, it is concluded that *a moderate degree of pathological traits is 'adaptive'* in the widest sense (D. K. Simonton, D. Schulberg, A. Storr, Art: 38-39., 40-42., 42-43.).

5.5. János Bolyai

It would seem that most of the above characteristics of eminent personalities and of their life circumstances do apply to János Bolyai (and in part to Farkas Bolyai too), such as intensive negative and positive experiences in childhood, mental problems in relative, many talents with excelling in each, ability to mobilize the whole personality, commitment to the task, subordinated 'deviant' style of experiencing and thinking, noble mentality. In János Bolyai's own criteria: "assignment" (competence, talent) and +determination" (unlimited enthusiasm) are needed for important achievement.

According to Root-Bernstein (p.103.) the role of mathematics is to *create order* where previously chaos seemed to reign and to extract structure and invariance from the midst of disarray and turmoil. In János Bolyai's case this role was perfectly fulfilled. Also, engagement with mathematics (including geometry) meant for him his "assignment" to improve the world and it denoted the meaning of his life.

5.6. Imre Hermann on Bolyai's creativity :

5.6.1. Imre Hermann on Bolyai

In his book on the 'Psychology of the birth of an idea' from 1945, Imre Hermann finds three groups of contributing factors in the background of János Bolyai's great

accomplishment: (a) inherited mathematical talent; (b) symbolically expressed personal conflicts (intensive attachment and separation conflict with mother, rivalry with father, fear of becoming confused); (c) specific cultural influences such as his education, science, visual observations, influences from Enlightment and the pre-Romantic Rococo period, and interests in genetic studies and in social movements. (The psychoanalyst did not mention sexuality as a central component in Bolyai!s achievements.)

5.6.2. Careful education

Hermann and all the other biographers emphasize János' exceptionally *careful education* by his father who realized how enthusiastic and talented his son was. (He was constantly inquisitive asking to be taught more and more.) As an objective study indicated (Ericson and Charned, 1994), supervised training at a very early age and maintained at a high daily level for more than a decade, is instrumental to facilitate elite performance. (There are inquiries into possible adaptive changes in the brain.)

5.6.3. Abstracting from visual experiences

Hermann points out that some *specific visual experiences* may have contributed to the emergence of the particular Bolyai type non-Eucledian geometry. János was exposed to the visual perceptual experience of the by him lengthily analyzed '*tractrix' (*trumpet) form and of the converse '*saddle'* shape, as well as of *tent-shapes* which he could see day by day while serving in the military. Also, from his early childhood on, Bolyai had the daily visual experience of the converse triangle shape of the tower-top of the Minorite church just in front of the windows of the Bolyai house. Further,

there were ‘tent’ like architectural *hyperbolic forms of stone or wood-built church towers* in the historical Hungarian landscapes and in Eastern Europe. For all these perceptions the sum of degrees of the angles in the curved ‘triangles’ was less than $180^{o.}$, like it is in the Bolyai type non-Eucledian geometry. Yet, the same visual experiences were available for a large segment of the population. The interesting thing is, therefore, that Bolyai differentiated and absorbed these isomorphic peripheral background perceptions which were stable and unnoticed by others. He may have analogically regrouped his visual experiences and used them as models for solving abstract problems. (Lobachevski may have been similarly exposed to and was influenced by visual experiences of onion-shapes in the Russian Orthodox churches; and in his version of geometry the sum of the angles is more than 180 degree).

5.6.4. Healing through clear thinking.

The intensity of Bolyai’s commitment to faultless thinking and his astute intolerance of any logical contradiction is seen by Hermann as a defence originating in a not necessarily conscious *fear of becoming schizophrenic* (thought disordered) on an inherited basis. Hermann finds similar *’schizoid-formalistic’ thinking style* in Bolyai’s and in David Hilbert’s productive thought processes: highly organized style and meticulous precision; striving for perfection and completeness; intolerance of contradiction; turning away from concrete practical perceptions to ideational concepts, with return to perception through using ’signs’ which, however, have no concrete meanings in themselves; further, omnipotence of thought; fighting against some possible end of the world notion or against a ’nothing’ (He. pp. 40-50.).

According to Stephen Strack (1999), normal and disordered individuals of the same personality type are essentially the same from the point of view of their basic trait make-ups. The major difference consists in the degree of flexibility and adaptability

(control) in using these traits, or, according to Ehrenwald's formulation, in the *ability to periodically integrate and mobilize the whole personality.*

6. THE SEARCH FOR THE FINAL CAUSES

Bolyai gave hints about his conjectures on Ultimate Reality. We shall group these hints into personal or feeling based assumptions and into more object-focused intellectual assumptions. A quite coherent worldview by Bolyai can be hypothetically extracted.

6.1. Feeling based assumptions about Ultimate Reality

6.1.1. Creating whichever new worlds just using a 'sign', gives the impression of a touch of numinous and magic level mastery and manipulation introduced into science.

6.1.2. The person who (like Bolyai himself) creates with using the appropriate signs and is not false or contradictory, can share with the Divine power.

6.1.3. Twenty years old, exuberant Bolyai had the genuine experience of having been blessed and *sharing with the Divine power.* However, open mysticism was not a possibility for a post-Enlightment stance scientist, as it was almost offered later to the quantum theorists. Therefore he had to modify and formulate his spiritual experiences differently. Bolyai wrote in the 'Üdvtan' that he would not like to 'boast', and that he does not want to appear as if he fantasized himself to be God's elect, but still, it would seem that he is the person „assigned", showing the 'talent and the competence', and „destined" to devote himself to work on the happiness of

mankind since having the ‘enthusiasm for the task’. He may have had the genuine feeling or experience that the meaning, or the call, of his life is that he is an ’elect’, but his conscience warned him against yielding to this unproven, doubtful, and for him antidemocratic and self-aggrandizing idea.

Having a strong conscience, Bolyai may have felt guilty for his experience of sharing with the Divine and he feared possible hubris or ego-inflation (feeling as if creating from nothing). He feared his tendency to self-aggrandizement which also clashed with his democratic political stance according to which all humans are of equal value. But despite these concerns Bolyai experienced that his wish to share with the divine was partially satisfied. No matter of his often tragic life, *Ultimate reality was generous to him* and gave him more gifts than to most of us. Mature Bolyai experienced that the ‘*highest spirit’ was ’enfolding’ before him.*

6.1.4. Farkas Bolyai wrote in the Arithmetica: “Where is that Moses who will smash the golden idol?” (He. 1845, 80.). János, who was intimate with the thoughts of his ‘father and teacher’, may have felt that he succeeded in throwing down a ‘golden idol’, Euclidean geometry, while also, at the same time, loyally saving the non-idol parts of it, allocating Euclidean geometry within the system of absolute geometry. Even though the awareness of Moses-like achievement may have been very embarrassing for János’ originally modest self-image and identity, he had to live in gratitude and with the unique overwhelming experience that *he was in contact with the tremendous Arch-power,* the cosmic arch-mother.

6.1.5. In the manuscript ’Reformation der Elemente der Mathematik’ quoted in Weszely (1981) Bolyai described the condition of the ’mathematician’: *The mathematician is freed from sensual inspiration, has the awareness of greatest and*

purest, highest happiness and of disciplined calmness. He tries his best to understand the final cause and the interconnection of everything. The aim of the mathematician is to know the 'highest spirit' Who („O") enfolds for him more and more, filling him with ecstasies and with sublime love for that highest spirit", the over-all present Ultimate Reality, and the final cause and 'essence' of all things.

6.1.6. Gratefulness may have lifted and protected Bolyai's morals in difficult times. If he wanted to protect this satisfaction which we call here "partially sharing", the only possible non-contradictory way was to maintain the style of generosity modeled upon Ultimate Reality. As Bolyai's autobiographical notes indicate, after in cases of rage the angry impulses stilled down, and he *managed to turn his would be justified hate into generosity* (like towards Gauss and towards his father). He maintained the dignity of somebody who has been a *sort of an elect.* ('Gratefulness' is an indication of 'human maturity' according to Imre Hermann, 1955, and more recently, McCullough et al., 2001.)

6.1.7. We could summarize Bolyai's personal attitude towards Ultimate Reality as *ambivalence with maintained gratitude, idealism, trust, and even ecstatic enthusiasm.* Amidst all the disappointments and depression, and the misunderstandings in his short-spirited home town, Bolyai maintained his basic experience of enthusiasm and harmony.

6.2. Bolyai's world view.

6.2.1. The close analogy between epigenesis and non-Euclidean geometry automatically suggests the possibility of a generalized epigenetic philosophical view about *Ultimate Reality as the arch-form for epigenesis* with unlimited variations.

This view shows similarity to the ’One and many’ problematic in metaphysics (Bedrij, 2000). In Bolyai it is treated as the epistemological pluri-univocal relationship between one parent and many offsprings. Ultimate Reality is the inherent source of everything, such as in the symbolic, abstract expression the ’parent’ space or the parent point are.

6.2.2. In Bolyai’s symbolization, Ultimate Reality is the Divine which can be seen or experienced in two forms, as the immanent universal law and as an immanently indwelling unfolding spirit.

6.2.3. Ultimate reality is unlimitedly rich in creating infinite numbers of new and possible worlds. The ’pleroma’ experience of the Stoics, of St. Paul, and of the Neo-Platonists was present in Bolyai, too.

6.2.4. Ultimate Reality is *present in all his ‘children’* through giving them the inheritance of the *encompassing common law,* Providence.

6.2.5. Arch-power Ultimate Reality is „amorphous”, but it is the “system” of permanent structural rules. *The ’whole world’ is an interconnected, ’perfect’, ’organized whole’* (Üdvtan, ref. He. 1945, 61.). It shows harmony.

6.2.6. Since Ultimate Reality uniformly penetrates everything, it is *infinitely tolerant towards ’all’ his ’children’ (*’spaces’, ’worlds’, persons) who are all of equal rank. Infinite spaces, and even ordinarily seen absurdities (such as triangles with less than 180° sum of angles, or social injustice) are possibilities and do not violate the harmony.

6.2.7. It goes with this neutrality and tolerance with lacking preference that *Ultimate Reality gives the possibility of happiness, but it does not provide or ensure it.*

Working on his happiness is left to man, who, applying some given rules (the appropriate 'i's), has to create the wished for new subsystem or world.

6.2.8. The intellectual 'sign' 'i' is a potential link to connect abstract (i.e. geometric) terms with existential beliefs about reality. *Using an 'i' man can perform the transformational act to create an endless number of new systems* as variations of the non-changing, all-encompassing 'absolute' system.

6.2.9. The 'i' seems to symbolize man's *bounded-bounding freedom* to transform the existing world through certain mediating rules and tools. Amorphous Ultimate Reality, or "highest Spirit", did not restrict the values and 'formulas' of the creating or modifying 'i'-s; but left this to man's deliberate choice. This is almost divine, for man can do everything, the only limits are the laws of nature.

6.2.10. Since non-Euclidean geometry reflects the general natural law to be studied and applied, geometry is a contribution to promote the welfare of mankind.

6.2.11. Bolyai shared the utopian optimism of his time. To improve the world, man has to construct and evaluate the goals and means on a scientific basis, for *only science can ensure happiness* through its expressing the harmony of the encompassing laws of Ultimate Reality. For Bolyai, like for today's social constructivists, reasoning, knowledge, intellect, are not passive mirroring functions, but are parts of the imagining actions and have metaphysical causative potential. Bolyai's reflections turn to anticipate post-modern 'social constructivism', particularly of Richard Rorty's and Ken Gergen's, but without their antirealism. Bolyai firmly believed in objective reality and in the power of knowledge. Through scientifically based creative action the human person is in continuity with the divine.

6.2.12. In Bolyai's close to postmodern mentality *reality* would seem to consist from 1./ *Ultimate Reality*, present in all his children through the laws of Providence; 2./ the interconnected concrete natural *laws*; 3./ possible and actual *worlds*; 4./ the *acts of creation* or creative transformations which latter can be continued by man, too; 5./ the creating and *transforming symbolic 'i'* functions which represent epigenetic processes and man's bounded freedom to give meaning and make structural changes through adding modifying rules; 6. the *planning and acting* function of man who is equipped with reason which connects him with the divine.

6.2.13. Space and Ultimate Reality itself, seem to be *expanding.* The emerging worlds multiply and 'burst away'. However, according to Bolyai, limiting components may enter; i.e. the laws of gravitation and the curvature of space.

6.2.14 . *'Evil' is logical 'contradiction';* falsehood, not a thing. It makes reasoning and creating 'impossible'. While concrete absurdities and bizarreness are 'understandable', abstract 'contradiction' is not, and it is therefore intolerable. (The evil spirit seems to be more against the universal laws of Providence than against the personal power of God.) *Logical procedure is an ethical requirement.* . (In the observation of Imre Hermann from 1929, logical thinking and conscience, resp. the superego, represent the ability to employ rules. They co-emerge in the individual development at around the age of five,)

6.2.15. 'Virtue' is 'logic', and *the final good is the objective 'truth'* which integrates man's teleological freedom with the deterministic universal law perspective and gives to man what Hermann calls 'bounding bounded' freedom. Objective truth makes science and the improvement of the world possible. In non-Euclidean geometry *any free creation brings forth a subsequently bounded, determined world* (i.e. He. 34-35.).

6.2.16. *'Ultimate Meaning'* involves a plurality of possible meanings and goals. It is originally *'uncertain' which world is the best* (He. Í845, 50.). The quest for objective truth via the social and the physical sciences helps us to evaluate existing and possible worlds and to construct sensible plans. This bounding-bounded view of freedom by Bolyai appears as congruent with Nobel-prize winner Richard Sperry's (1985) suggestion that in order to construct workable ethical and social rules we need an independent 'science of bidirectional causality', the study of interaction of 'up-down mental' and of 'bottom-up deterministic' causation.

6.2.17. Regarding religious meaning, it would seem that *Bolyai's religion* was Stoic, philosophical and immanent, with inclination to function with an *ecstatic cosmic consciousness.* (All existence and events are "equal" without hierarchy; R.M. Bucke, 1923; Abraham Maslow, 1969; Ken Wilber, 1995.) Also, Bolyai found solace considering that *the imperfections of the world do not violate the perfect harmony* provided by Ultimate Reality which can be expressed through mathematics. If we want to become happy, this can be gradually achieved through the generations with human means, education and science.

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